Using a GPS to compute True Airspeed

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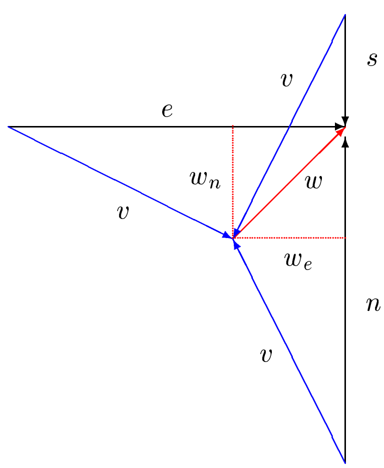
This note describes two ways to use GPS groundspeed readings to compute true airspeed. The ideas are not original and have been around for some time. Both techniques involve flying three perpendicular legs and measuring the groundspeed on each leg. In the first method, one flies perpendicular tracks, in the other one flies perpendicular headings. The first technique is due to David Fox and was published in *Kitplanes* in 1995. The second technique has first been proposed by Gregory V. Lewis of the National Test Pilot School. In this note I'll derive the airspeed formulas for both techniques. The computations can be done with most any pocket calculator. Better yet, you can just go to the EAA Chapter 62 web site (`http://www.eaa62.org/technotes/speed.htm`) and use the online calculator that implements the perpendicular heading method.

Flying Perpendicular Tracks

The following figure illustrates the situation.



The arrows labeled n (north), s (south), and e (east) are the speed vectors of the three ground tracks. These tracks are actually the result of flying with speed v on a heading that compensates for the wind w. I've drawn the speed vectors so they all end up at the same point even through nobody would fly the patterns that way. But it does not really matter - things are the same if you move the tracks around on the plane - it's only the direction that matters.

Drawing things in this way makes the analysis a bit simpler. For instance, it is easy to see that $s + w_n = n - w_n$ where w_n is the component of the wind that blows north, and w_e is the component of the wind that blows east. The quantities that we know in this picture are n, e, s. What we'd like to know is v, the true airspeed, and w_e, w_n which give us the strength and the direction of the wind. From $s + w_n = n - w_n$ we find

$$w_n = \frac{n - s}{2} \tag{1}$$

Next, we appeal to Pythagoras and get the two equations (you find the right triangles)

$$w_n^2 + (e - w_e)^2 = v^2 \tag{2}$$
$$w_e^2 + (s + w_n)^2 = v^2 \tag{3}$$

From equations (2,3) we get (4) which simplifies to (5) and yields a value for w_e (7):

$$w_n^2 + (e - w_e)^2 = w_e^2 + (s + w_n)^2 \tag{4}$$
$$e^2 - 2ew_e = s^2 + 2sw_n \tag{5}$$
$$w_e = \frac{e^2 - s^2 - 2sw_n}{2e} \tag{6}$$
$$= \frac{e^2 - sn}{2e} \tag{7}$$

where equation (7) follows from (6) by substituting (1) for w_n. The rest is now pretty simple, starting with equation (2) and using (1,7), we have

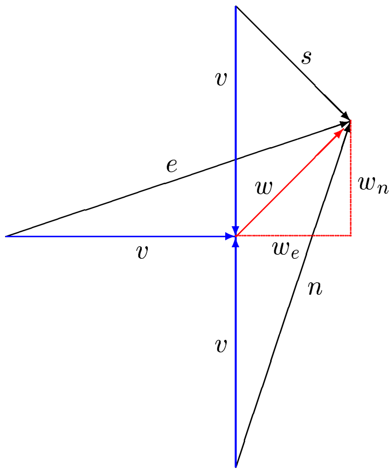
$$v^2 = w_n^2 + (e - w_e)^2 \tag{8}$$
$$= \frac{(n - s)^2}{4} + \left(\frac{e^2 + sn}{2e}\right)^2 \tag{9}$$
$$= \frac{n^2 + s^2 + e^2 + \frac{s^2 n^2}{e^2}}{4} \tag{10}$$

If you care about the wind speed and direction, you can get the former as $w = \sqrt{w_e^2 + w_n^2}$ and the latter as $\text{atan}(w_e/w_n)$. Summarizing all of this we have:

$$
\boxed{\begin{array}{rcll} c & = & n^2 + s^2 + e^2 + \frac{s^2 n^2}{e^2} & \text{Temporary} \\ v & = & \frac{1}{2}\sqrt{c} & \text{True airspeed} \\ w & = & \frac{1}{2}\sqrt{c - 4ns} & \text{Wind speed} \end{array}}
$$

Flying Perpendicular Headings

Flying three legs with constant heading may be easier to do, but the formulas get a slight bit more tricky. The speed vectors look as follows.



From the groundspeeds n, e, and s we want to compute v and w_e and w_n. We start with three right triangles, each made up of (i) the forward speed plus head or tail wind component, (ii) the cross wind component, and (iii) the groundspeed:

$$
\begin{aligned}
e^2 &= (v + w_e)^2 + w_n^2 && (11) \\
s^2 &= (v - w_n)^2 + w_e^2 && (12) \\
n^2 &= (v + w_n)^2 + w_e^2 && (13)
\end{aligned}
$$

Adding and subtracting these equations, we get

$$
\begin{aligned}
n^2 + s^2 &= 2(v^2 + w_e^2 + w_n^2) && (14) \\
n^2 - s^2 &= 4vw_n && (15) \\
e^2 - \frac{n^2 + s^2}{2} &= 2vw_e && (16)
\end{aligned}
$$

To make things a bit more readable, we introduce three new variables that are all defined in terms of known quantities:

$$
\begin{aligned}
c_0 &= \frac{n^2 - s^2}{2} && (17) \\
c_1 &= \frac{n^2 + s^2}{2} && (18) \\
c_2 &= e^2 - c_1 && (19)
\end{aligned}
$$

With these definitions the equations (14,15,16) become

$$
\begin{aligned}
c_1 &= v^2 + w_e^2 + w_n^2 && (20) \\
w_n &= \frac{c_0}{2v} && (21) \\
w_e &= \frac{c_2}{2v}. && (22)
\end{aligned}
$$

Substituting (21,22) into (20) gives us

$$
c_1 v^2 = v^4 + c_2^2/4 + c_0^2/4. \qquad (23)
$$

This looks worse than it is because with $V = v^2$ this turns into a simple quadratic equations

$$
V^2 - c_1 V + c_2^2/4 + c_0^2/4 = 0 \qquad (24)
$$

that can be solved for V as

$$
V_{1,2} = \frac{c_1 \pm \sqrt{c_1^2 - c_2^2 - c_0^2}}{2}
$$

or

$$
v_{1,2} = \sqrt{\frac{c_1 \pm \sqrt{c_1^2 - c_2^2 - c_0^2}}{2}}
$$

So why do we get two solutions? It turns out that one solution is the true airspeed we are looking for and the other solution is the wind speed. So altogether we get

$$
\boxed{\begin{array}{rcll} c_0 & = & (n^2 - s^2)/2 & \text{Temporary} \\ c_1 & = & (n^2 + s^2)/2 & \dots \\ c_2 & = & e^2 - c_1 & \dots \\ v & = & \sqrt{\frac{c_1 + \sqrt{c_1^2 - c_2^2 - c_0^2}}{2}} & \text{True airspeed} \\ w & = & \sqrt{\frac{c_1 - \sqrt{c_1^2 - c_2^2 - c_0^2}}{2}} & \text{Wind speed} \end{array}}
$$

As before, the wind direction can be computed as atan(w_e/w_n), using (21) and (22).